The question in the seminar

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To: Liu, Tong <tongliu@purdue.edu>;

Dear Prof. Liu,

I think the question should be stated as this:

If S is a perfectoid ring in the sense of [BMP, 3.5] with $\pi \in S$ as in the definition, and assume $S[1/\pi]$ is a field that contains S as open subring(i.e. with the π-adic topology), does the topology of S induced by a norm (of rank one)?

The answer is yes, we can construct

$$|\cdot| : S \to \mathbb{R}_{\geq 0} : |s| = 2^{-n_s} \text{ where } n_s = -\max\{n \in \mathbb{Z} | s \in \pi^n S\}$$

can check this is a Banach norm on S as in page 11 of

http://www.math.uni-bonn.de/people/scholze/Berkeley.pdf

Berkeley lectures on p-adic geometry

www.math.uni-bonn.de

4 CONTENTS Preface This is a revised version of the lecture notes for the course on p-adic geometry given by P. Scholze in Fall 2014 at UC Berkeley.

Actually, any Hausdorff Huber ring is metrizable by page 4 of https://www.mathi.uni-heidelberg.de/~G.QpAsPi1geom/manuscripts/AS.pdf. And in our case, we have the underlying \Q_p vector space of S[1/\pi] is metrizable iff it has a countable basis at 0 by a general theory of topological vector space, and the basis is {g^nS} and the construction is exactly as above given by Scholze.

Best,

Heng